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A new approach to m-topological spaces

Sk. Nazmul[1], Mathematics, Bankura University, India

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Abstract

In the present paper, we have introduced a notion of multi topological space (m-topological space in short) using a new approach and detailed topological properties are studied. We have also noticed that the constant mapping is not m-continuous and projection mappings are not m-open in this m-topological spaces. For this, we have defined enriched m-topological spaces and the continuity of constant mapping, openness of projection mappings are established.

Keywords: *Multi sets, multi topologies, multi subspace, m-continuity, m-openness.*

1. Introduction

Uncertainty is the most prevalent aspects in the natural occurrence of events and several theories such as probability theory, fuzzy set theory, rough set theory, multi set theory, soft set theory were developed to deal with it. Multisets (m-sets in short), first suggested by N. G. de Bruijn (1983) in a private communication to D. E. Knuth, is an important generalization of classical set theory which has emerged by violating a basic property of classical sets that an element can belong to a set only once. Owing to its aptness, it has replaced a variety of terms viz. list, heap, bunch, bag, sample, weighted set, occurrence set and fireset used in different contexts but conveying synonimity with m-set. These sets are very useful structures arising in many areas of mathematics and computer science. Many authors like Yager (1986), Miyamoto (2003), Hickman (1980), Blizard (1989), Girish and John (2009-2012), Hallez et al. (2009) etc. have studied the properties of m-sets which is extended to multi groups by Nazmul et al. (2013). Afterward, several authors have also generalized the notion of m-sets in the setting of soft sets to form soft multisets (Alkhazaleh & Salleh, 2011), (Babitha & John, 2013), (Majumdar & Samanta, 2012), multi soft sets (Herawan & Mustafa, 2009), soft multi groups (Nazmul & Samanta, 2015), soft multi topological space, Connectedness and Compactness on soft multi topological spaces (Tokat & Osmanoglu, 2013). As a continuation of this and in developing the algebraico topological structures, it is natural to study the behaviour of topological structures in m-set settings. Grish and John (2009) have introduced the notion of m-set topology (m-topology in short) as a collection of m-sets satisfying some conditions and studied some of its properties. As a continuation of this it is natural to investigate the behaviour of algebraico topological structures in this setting. The development of these structures requires the continuity of product space, but we have seen that, the m-topology thus defined is less user friendly for handling the continuity of product m-set spaces. Keeping this in view we have tried to define m-topological space in an approach and detailed topological properties are studied. We have also noticed that projection mappings are m-continuous but not m-open and constant mapping is not m-continuous in general. For this, we have introduced the concept of enriched m-topological space where projection mappings are m-open and constant mapping is m-continuous.

[1] Corresponding author (sk.nazmul_math@yahoo.in)

2. Preliminaries

This section consists of some definitions and results of m-sets and m-topologies which will be used in the main works of the paper. Unless otherwise stated, X will be assumed to be an initial universal set and $\mathbb{N}$ represents the set of all non-negative integers.

Multi sets (or m-sets):

Definition 2.1. (Grirish & John, 2009) An m-set M drawn from the universal set X is represented by a Count function C_M defined as $C_M : X \to \mathbb{N}$, where $\mathbb{N}$ represents the set of non-negative integers.

Here $C_M(x)$ is the number of occurrence of the element x in the m-set M. The presentation of the m-set M drawn from $X = \{x_1, x_2, \ldots, x_n\}$ will be as $M = \{x_1 / m_1, x_2 / m_2, \ldots, x_n / m_n\}$ where m_i is the number of occurrences of the element $x_i, i = 1, 2, \ldots, n$ in the m-set M.

Also here for any positive integer w, $[X]^w$ is the set of all m-sets whose elements are in X such that no element in the m-set occurs more than w times and $[X]^\infty$ is the set of all m-sets whose elements are in X such that there is no limit on the number of occurrences of an element in a m-set. As in \cite{GJ2}, $[X]^w$ and $[X]^\infty$ will be referred to as m-set spaces. *MS(X)* denotes the set of all m-sets drawn from X.

Definition 2.2. (Grirish & John, 2009) Let M_1 and M_2 be two m-sets drawn from a set X. Then M_1 is said to be sub m-set of M_2 if $C_{M_1}(x) \le C_{M_2}(x), \forall x \in X$. This relation is denoted by $M_1 \subseteq M_2$. M_1 is said to be equal to M_2 if $C_{M_1}(x) = C_{M_2}(x), \forall x \in X$. It is denoted by $M_1 = M_2$.

Definition 2.3. (Grirish & John, 2009) Let $\{M_i; i \in I\}$ be a nonempty family of m-sets drawn from the set X. Then

(a) Their *Intersection*, denoted by $\bigcap_{i \in I} M_i$ where $C_{\bigcap_{i\in I} M_i}(x) = \wedge_{i \in I} C_{M_i}(x), \forall x \in X$.

(b) Their *Union*, denoted by $\bigcup_{i \in I} M_i$ where $C_{\bigcup_{i\in I} M_i}(x) = \bigvee_{i \in I} C_{M_i}(x), \forall x \in X$.

(c) The *Complement* of any m-set M_i in $[X]^w$, denoted by M_i^c where $C_{M_i^c}(x) = w - C_{M_i}(x), \forall x \in X$.

Definition 2.4. (Nazmul et al. 2013) Let X and Y be two nonempty sets and $f : X \to Y$ be a mapping. Then

(i) the *image* of a m-set $M \in [X]^w$ under the mapping f is denoted by $f(M)$, where

$$C_{f(M)}(y) = \begin{cases} \vee_{f(x)=y} C_M(x) & \text{if } f^{-1}(y) \neq \phi \\ 0 & \text{otherwise} \end{cases}$$

(ii) the *inverse image* of a m-set $N \in [Y]^w$ under the mapping f is denoted by $f^{-1}(N)$, where $C_{f^{-1}(N)}(x) = C_N[f(x)]$.

Proposition 2.5. (Nazmul et al. 2013) Let *X*, *Y* and *Z* be three nonempty sets and $f: X \to Y$, $g: Y \to Z$ be two mappings. If $M_i \in [X]^w, N_i \in [Y]^w, i \in I$ then

(i) $M_1 \subseteq M_2 \Rightarrow f(M_1) \subseteq f(M_2)$.

(ii) $f[\bigcup_{i\in I} M_i] = \bigcup_{i\in I} f[M_i]$.

(iii) $N_1 \subseteq N_2 \Rightarrow f^{-1}(N_1) \subseteq f^{-1}(N_2)$.

(iv) $f^{-1}[\bigcup_{i\in I} M_i] = \bigcup_{i\in I} f^{-1}[M_i]$.

(v) $f^{-1}[\bigcap_{i\in I} M_i] = \bigcap_{i\in I} f^{-1}[M_i]$.

(vi) $f(M_i) \subseteq N_j \Rightarrow M_i \subseteq f^{-1}[N_j]$.

(vii) $g[f(M_i)] = [gf](M_i)$ and $f^{-1}[g^{-1}(N_j)] = [gf]^{-1}(N_j)$.

Definition 2.6. (Nazmul et al. 2013) Let $P \subseteq X$. Then for each $n \in \mathbb{N}$, we define a m-set nP over *X*, where $C_{nP}(x) = n, \forall x \in P$. This m-sets are called *level m-sets*.

Definition 2.7. (Nazmul et al. 2013) **A**n m-set containing only one element *x* of *n* times is called a *singleton m-set* and it is denoted by n_x.

Definition 2.8. (Grirish & John, 2009) Let *X* be a non-empty set and *M* be an m-set drawn from *X*. If $C_M(x) = n$, then we say $x \in^n M$.

Definition 2.9. (Grirish & John, 2012)Let $M \in [X]^w$ be a m-set and $P^*(M)$ be the collection of all sub m-sets of *M*. A sub collection τ of $P^*(M)$ is said to be a multiset topology (m-topology in short) on *M* if

(i) $M, \phi \in \tau$;
(ii) the intersection of any two m sets in τ belongs to τ ;
(iii) the union of any number of m sets in τ belongs to τ .

The pair (M, τ) is called a m-topological space on *M*.

Definition 2.10. (Grirish & John, 2012) Let (M, τ) be an m-topological space and *N* is a sub m-set of *M*. The collection $\tau_N = \{N \cap U : U \in \tau\}$ is an m-topology on *N*, called the subspace m-topology.

Definition 2.11. (Grirish & John, 2012) Let *M* and *N* be two m-topological spaces. The m-set function $f: M \to N$ is said to be continuous if for each open sub m-set *V* of *N*, the m-set $f^{-1}(V)$ is an open sub m-set of *M*, where $f^{-1}(V)$ is the m-set of all points x/m in *M* for which $f(m/x) \in^n V$ for some *n*.

3. M-topological Space and some of its properties

In this section, we have define m-topological space using another approach and detailed topological properties are studied. Enriched m-topological space is also introduced and the continuity, openness of projection mappings, constant mapping are studied.

Definition 3.1. Let X be a non-empty set and $P({}_wX)$ be the collection of all sub m-sets of ${}_wX$. A sub collection τ of $P({}_wX)$ is said to be a multi topology (m-topology in short) on ${}_wX$ if

(i) ${}_0X, {}_wX \in \tau$;
(ii) the intersection of any two m sets in τ belongs to τ ;
(iii) the union of any number of m sets in τ belongs to τ .

The pair $({}_wX, \tau)$ is called a m-topological space on ${}_wX$ and the members of τ are said to be open sub m-sets of $({}_wX, \tau)$ or simply τ -open sub m-sets.

Example 3.2. Let X be a non-empty set. Let $\mathcal{I} = \{{}_0X, {}_wX\}$ and $\mathcal{D} = P({}_wX)$. Then $\mathcal{I}$ and $\mathcal{D}$ are m-topologies on ${}_wX$.

$\mathcal{I}$ is said to be indiscrete and $\mathcal{D}$ is discrete m-topology on ${}_wX$.

Definition 3.3. Let $({}_wX, \tau)$ be a m-topological space. A sub m-set F of ${}_wX$ is said to be closed sub m-set of $({}_wX, \tau)$ if $F^c \in \tau$.

Proposition 3.4. If $\mathcal{C}$ be the family of all closed sub m-sets of $({}_wX, \tau)$, then

(i) ${}_0X, {}_wX \in \mathcal{C}$;
(ii) the union of any two members in $\mathcal{C}$ belongs to $\mathcal{C}$;
(iii) the intersection of any number of members in $\mathcal{C}$ belongs to $\mathcal{C}$.

Conversely if $\mathcal{C}$ be the family of sub m-sets of ${}_wX$ satisfying the above three conditions and if $\tau = \{U \subseteq {}_wX : U^c \in \mathcal{C}\}$, then τ is an m-topology on ${}_wX$ such that $\mathcal{C}$ is the family of all closed sub m-sets of $({}_wX, \tau)$.

Proof: Proof is straightforward.

Definition 3.5. A sub collection $\mathcal{B}$ of $P({}_wX)$ is said to be an m-basis of an m-topology τ on ${}_wX$ if every member of τ can be expressed as the union of some members of $\mathcal{B}$.

Proposition 3.6. Let $({}_wX, \tau)$ be a m-topological space. A sub collection $\mathcal{B}$ of τ forms a base of τ iff $\forall U \in \tau$ and $\forall x \in^p U, \exists V_p \in \mathcal{B}$ such that $x \in^p V_p \subseteq U$.

Proof: Let $\mathcal{B}$ be an m-base of τ and $U \in \tau$ such that $x \in^p U$. Then U can be expressed as the union of some members of $\mathcal{B}$. So, $\exists V_p \in \mathcal{B}$ such that $x \in^p V_p \subseteq U$. [if $\exists y (\neq x) \in X$ such that $C_{V_p}(y) > C_U(y)$, then $\cup_{x \in^p U} V_p \supset U$.] Therefore, the given condition is satisfied.

Conversely let the given condition be satisfied. Let $U \in \tau$ and $x \in^p U$. Then by the given condition, $\exists V_p \in \mathcal{B}$ such that $x \in^p V_p \subseteq U$. Now $p_x \subseteq V_p \subseteq U, \forall x \in^p U \Rightarrow U = \cup_{x\in^p U}\{p_x\} \subseteq \cup_{x\in^p U} V_p \subseteq U$. Thus, $U = \cup_{x\in^p U} V_p$ and hence U can be expressed as the union of some members of $\mathcal{B}$. Therefore, $\mathcal{B}$ is an m-base of τ.

Proposition 3.7. Let $X = \mathbb{R}$, the set of all real numbers, $0 < p(\in \mathbb{N}) \le w$ and $\mathcal{B} = \{U \subseteq_w X : \forall x \in^p U, \exists \delta > 0 \text{ such that } {}_p[(x-\delta, x+\delta)] \subseteq U\}$. Then $\mathcal{B}$ forms an m-base of an m-topology on ${}_w X$.

Proof: Since ${}_0 X \subseteq_w X$ and there is no $x \in^p {}_0 X, p > 0$, the above condition is trivially true and hence ${}_0 X \in \mathcal{B}$. Again since ${}_w X \subseteq_w X$ and $x \in^w {}_w X, \forall x \in X$, it follows that ${}_w[(x-1, x+1)] \subseteq_w X$ and hence ${}_w X \in \mathcal{B}$.

Next let $U_1, U_2 \in \mathcal{B}$ and $x \in^p (U_1 \cap U_2)$. Then without loss of generality, we assume that $x \in^p U_1$ and $x \in^q U_2$ for $q \ge p$. Since $U_1, U_2 \in \mathcal{B}, \exists \delta_1 > 0, \delta_2 > 0$ such that ${}_p[(x-\delta_1, x+\delta_1)] \subseteq U_1$ and ${}_q[(x-\delta_2, x+\delta_2)] \subseteq U_2$ for $q \ge p$. Let $\delta = \delta_1 \wedge \delta_2$ and $U_3 = {}_p[(x-\delta, x+\delta)]$. Then $x \in^p U_3 \subseteq (U_1 \cap U_2)$.

Thus, by Proposition 3.6., we have $\mathcal{B}$ forms an m-base of an m-topology on ${}_w X$.

Definition 3.8. The m-topology induced by the m-basis defined in Proposition 3.7., is called the usual m-topology or metric m-topology on ${}_w\mathbb{R}$..

Definition 3.9. Let $({}_w X, \tau)$ be an m-topological space. A sub m-set N of ${}_w X$ is said to be an m-neighbourhood (m-nbd in short) of a sub m-set M of ${}_w X$ if $\exists U \in \tau$ such that $M \subseteq U \subseteq N$. In particular if $M = p_x$, then N is said to be m-nbd of p_x.

Proposition 3.10. A sub m-set U of ${}_w X$ is open iff U is an m-nbd of $p_x, \forall x \in^p U$.

Proof: First let $U \in \tau$ and $x \in^p U$. Then $x \in^p U \subseteq U$ and hence U is an m-nbd of $p_x, \forall x \in^p U$.

Conversely let the given condition be satisfied.

Let $x \in^p U$. Then by the given condition, $\exists V_{p_x} \in \tau$ such that $x \in^p V_{p_x} \subseteq U$. So, $p_x \subseteq V_{p_x} \subseteq U, \forall x \in^p U \Rightarrow U = \cup_{x\in^p U}\{p_x\} \subseteq \cup_{x\in^p U} V_{p_x} \subseteq U$. Thus $U = \cup_{x\in^p U} V_{p_x} \in \tau$ since each $V_{p_x} \in \tau$.

Proposition 3.11. Let $({}_w X, \tau)$ be an m-topological space and Y be a non-empty subset of X. Then $\tau_{w'Y} = \{U \cap_{w'} Y : U \in \tau\}$ is an m-topology on ${}_{w'} Y, \forall w' \le w$.

Proof: Since ${}_0X, {}_wX \in \tau,$ it follows that ${}_0X \cap_{w'} Y = {}_0Y \in \tau_{w'Y}$ and ${}_wX \cap_{w'} Y = {}_{w'}Y \in \tau_{w'Y}.$ Let $\{V_i, i \in \Delta\}$ be any collection of members of $\tau_{w'Y}$ and $V = \cup_{i\in\Delta} V_i.$ Since $V_i \in \tau_{w'Y}, \exists U_i \in \tau$ such that $V_i = U_i \cap_{w'} Y.$ Thus, $V = \cup_{i\in\Delta} V_i = \cup_{i\in\Delta}[U_i \cap_{w'} Y] = [\cup_{i\in\Delta} U_i] \cap_{w'} Y \in \tau_{w'Y}.$

Finally let $V_1, V_2 \in \tau_{w'Y}$ and $V = V_1 \cap V_2.$ Since $V_1, V_2 \in \tau_{w'Y}, \exists U_1, U_2 \in \tau$ such that $V_1 = U_1 \cap_{w'} Y$ and $V_2 = U_2 \cap_{w'} Y.$ So, $V = [U_1 \cap U_2] \cap_{w'} Y \in \tau_{w'Y}.$

Therefore, $\tau_{w'Y}$ is an m-topology on ${}_{w'}Y, \forall w' \leq w.$

Definition 3.12. Let $({}_wX, \tau)$ be an m-topological space and Y be a non-empty subset of $X.$ Then by Proposition 3.11., $\tau_{w'Y} = \{U \cap_{w'} Y : U \in \tau\}, w' \leq w$ is an m-topology on ${}_{w'}Y.$ This m-topology is called an m-relative topology on ${}_{w'}Y$ and $({}_{w'}Y, \tau_{w'Y})$ is called an m-subspace of m-topological space $({}_wX, \tau).$

Proposition 3.13. Let $({}_wX, \tau)$ be an m-topological space and Y be a non-empty subset of X and $w' \leq w.$ Then

(i) $P \in [Y]^{w'}$ is closed in $({}_{w'}Y, \tau_{w'Y})$ iff $P = Q \cap_{w'} Y,$ where Q is closed in $({}_wX, \tau);$

(ii) if $\mathcal{B}$ be an m-base of $\tau,$ then $\mathcal{B}_{w'Y} = \{U \cap_{w'} Y : U \in \tau\}$ is an m-base of $\tau_{w'Y}.$

Proof: (i) Let P be any closed m-set of $({}_{w'}Y, \tau_{w'Y}).$ Then $P^c \in \tau_{w'Y}$ and hence $\exists U \in \tau$ such that $P^c = U \cap_{w'} Y.$ Now,

$$\begin{aligned}
\text{P} &= (P^c)^c = (U \cap_{w'} Y)^c = {}_{w'}Y \cap (U \cap_{w'} Y)^c_\tau = {}_{w'}Y \cap [U^c_\tau \cup ({}_{w'}Y)^c_\tau] \\
&= [{}_{w'}Y \cap U^c_\tau] \cup [({}_{w'}Y)^c_\tau \cap_{w'} Y] = [{}_{w'}Y \cap U^c_\tau] \cup {}_0Y = [{}_{w'}Y \cap U^c_\tau] \\
&= {}_{w'}Y \cap Q, \text{where } Q \text{ is closed m-set in } ({}_wX, \tau).
\end{aligned}$$

(ii) Let $V \in \tau_{w'Y}$ and $x \in^p V.$ Since $V \in \tau_{w'Y},$ it follows that $\exists U \in \tau$ such that $V = U \cap_{w'} Y.$ Again since $\mathcal{B}$ is an m-base of τ and $x \in^p U,$ it follows that $\exists W \in \mathcal{B}$ such that $x \in^p W \subseteq U.$ Then $[W \cap_{w'} Y] \in \mathcal{B}_{w'Y}$ and $x \in^p [W \cap_{w'} Y] \subseteq [U \cap_{w'} Y] = V.$

Therefore, $\mathcal{B}_{w'Y} = \{U \cap_{w'} Y : U \in \tau\}$ is an m-base of $\tau_{w'Y}.$

Definition 3.14. Let X, Y be two non-empty sets, τ, ν be two m-topologies on ${}_wX, {}_{w'}Y$ respectively, $w' \leq w$ and $f : X \to Y$ be a mapping. The image of τ and preimage of ν under f are denoted by $f(\tau)$ and $f^{-1}(\nu)$ respectively, and is defined by

(i) $f(\tau) = \{N \in [Y]^{w'} : f^{-1}(N) \in \tau\};$

(ii) $f^{-1}(\nu) = \{f^{-1}(N) : N \in \nu\}.$

Proposition 3.15. Let X, Y be two non-empty sets, τ, ν be two m-topologies on ${}_wX, {}_{w'}Y$ respectively, $w' \leq w$ and $f : X \to Y$ be a mapping. Then

(i) $f^{-1}(\nu)$ is an m-topology on ${}_{w'}X$;
(ii) $f(\tau)$ is an m-topology on ${}_wY$.

Proof: (i) Since $f^{-1}({}_0Y)] = {}_0X, f^{-1}({}_{w'}Y)] = {}_{w'}X$ it follows that ${}_0X, {}_{w'}X \in f^{-1}(\nu)$.

Next let $M_1, M_2 \in f^{-1}(\nu)$. Then there exist $N_1, N_2 \in \nu$ such that $f^{-1}(N_1) = M_1$ and $f^{-1}(N_2) = M_2$. Now, $M_1 \cap M_2 = f^{-1}(N_1) \cap f^{-1}(N_2) = f^{-1}(N_1 \cap N_2) \in f^{-1}(\nu)$.

Again let $M_i, i \in \Delta$ be any collection of members of $f^{-1}(\nu)$. Then there exist $N_i \in \nu, i \in \Delta$ such that $f^{-1}(N_i) = M_i, i \in \Delta$. Now, $\tilde{\cup}_{i\in\Delta}(M_i) = \tilde{\cup}_{i\in\Delta}[f^{-1}(M_i)] = f^{-1}[\cup_{i\in\Delta}(M_i)] \in f^{-1}(\nu)$. Therefore, $f^{-1}(\nu)$ is an m-topology on ${}_{w'}X$.

(iii) Since $f^{-1}({}_0Y) = {}_0X \in \tau$ and $f^{-1}({}_wY) = ({}_wX) \in \tau$, it follows that ${}_0Y, {}_wY \in f(\tau)$. Next let $N_1, N_2 \in f(\tau)$. Then $f^{-1}(N_1), f^{-1}(N_2) \in \tau$.

Now, $f^{-1}(N_1 \cap N_2) = [f^{-1}(N_1) \cap f^{-1}(N_2)] \in \tau$ and hence $(N_1 \cap N_2) \in f(\tau)$.

Again let $\{N_i, i \in \Delta\}$ be any collection of members of $f(\tau)$. Then $f^{-1}(N_i) \in \tau, \forall i \in \Delta$. Now, $f^{-1}[\cup_{i\in\Delta}(N_i)] = \cup_{i\in\Delta}[f^{-1}(N_i)] \in \tau$ and hence $\cup_{i\in\Delta}(N_i) \in f(\tau)$. Therefore $f(\tau)$ is an m-topology on ${}_wY$.

Definition 3.16. Let $({}_wX, \tau)$ and $({}_wY, \nu)$ be two m-topological spaces. The mapping $f : ({}_wX, \tau) \to ({}_wY, \nu)$ is said to be

(i) m-continuous if $f^{-1}(N) \in \tau, \forall N \in \nu$;
(ii) m-open if $f(M) \in \nu, \forall M \in \tau$.

Proposition 3.17. Let $({}_wX, \tau)$ and $({}_wY, \nu)$ be two m-topological spaces. A mapping $f : ({}_wX, \tau) \to ({}_wY, \nu)$ is m-continuous iff $\forall x \in X$ and $\forall N \in \nu$ such that $f(x) \in^p N$, there exists $M \in \tau$ such that $x \in^p M$ and $f(M) \subseteq N$.

Proof: Let $f : ({}_wX, \tau) \to ({}_wY, \nu)$ be an m-continuous mapping. Let $x \in X$ and $N \in \nu$ such that $f(x) \in^p N$. Then $x \in^p f^{-1}N$. Since f is m-continuous and $N \in \nu$, it follows that $f^{-1}(N) \in \tau$. Let $M = f^{-1}(N)$. Then $x \in^p M$ and $f(M) = f[f^{-1}(N)] \subseteq N$. Therefore the given condition is satisfied.

Conversely let the given condition be satisfied.

Let $N \in \nu$ and $x \in^p f^{-1}(N)$. Then $f(x) \in^p N$ and hence by the given condition, $\exists M_{p_x} \in \tau$ such that $x \in^p M_{p_x}$ and $f(M_{p_x}) \subseteq N$. Then $M_{p_x} \subseteq f^{-1}(N)$. Now $f^{-1}(N) = \cup_{x \in^p f^{-1}(N)} \{p_x\} \subseteq \cup_{x \in^p f^{-1}(N)} M_{p_x} \subseteq f^{-1}(N)$. Thus $f^{-1}(N) = \cup_{x \in^p f^{-1}(N)} M_{p_x} \in \tau$, since each $M_{p_x} \in \tau$. Therefore, the mapping $f : ({}_w X, \tau) \to ({}_w Y, \nu)$ is m-continuous.

Proposition 3.18. Let $({}_w X, \tau)$ be an m-topological space. Then the identity mapping $f : ({}_w X, \tau) \to ({}_w X, \tau)$ is m-continuous.

Proof: Proof is straightforward.

Remark 3.19. Let $({}_w X, \tau), ({}_w Y, \nu)$ be two m-topological spaces. Then the constant mapping $f : ({}_w X, \tau) \to ({}_w Y, \nu)$ where $f(x) = y_0$ (a fixed element of Y), $\forall x \in X$ is not m-continuous in general which shows the following example.

Example 3.20. Let $X = \{x_1, x_2, x_3\}, Y = \{y_1, y_2, y_3, y_4\}$ be two non-empty sets, $w = 5, \tau = \{{}_0 X, \{2/x_1, 3/x_3\}, {}_5 X\}$, and $\nu = \{{}_0 Y, \{2/y_1, 3/y_2, 1/y_3, 4/y_4\}, {}_5 Y\}$. Then τ, ν are m-topologies on ${}_5 X$ and ${}_5 Y$ respectively. Now define a mapping $f : ({}_w X, \tau) \to ({}_w Y, \nu)$ where $f(x) = y_4, \forall x \in X$. Then $N = \{2/y_1, 3/y_2, 1/y_3, 4/y_4\} \in \nu$ but $f^{-1}(N) = \{4/x_1, 4/x_2, 4/x_3\} \notin \tau$. Therefore, the constant mapping f is not m-continuous.

Example 3.21. Let X be a non-empty set and $P({}_w X)$ be the collection of all sub m-sets of ${}_w X$. A sub collection τ of $P({}_w X)$ is said to be an enriched m-topology on ${}_w X$ if

(i) ${}_p X \in \tau, \forall p (\in \mathbb{N}) \leq w$;
(ii) the intersection of any two members in τ belongs to τ ;
(iii) the union of any number of members in τ belongs to τ .

The pair $({}_w X, \tau)$ is called an enriched m-topological space on ${}_w X$.

Proposition 3.22. Let $({}_w X, \tau), ({}_w Y, \nu)$ be two m-topological spaces. If $({}_w X, \tau)$ is enriched m-topological space, then the constant mapping $f : ({}_w X, \tau) \to ({}_w Y, \nu)$ where $f(x) = y_0$ (a fixed element of Y), $\forall x \in X$ is m-continuous.

Proof: Let $N \in \nu$. Now for any $x \in X$,

$$C_{f^{-1}(N)}(x) = C_N[f(x)] = \begin{cases} 0 & \text{if } C_N(y_0) = 0 \\ k & \text{if } C_N(y_0) = k \end{cases}$$

So, $f^{-1}(N) = {}_0 X$ or $f^{-1}(N) = {}_k X$. Since τ is enriched m-topology on ${}_w X$, it follows that ${}_0 X, {}_k X \in \tau$. Therefore, the constant mapping f is m-continuous.

Proposition 3.23. Let $({}_wX,\tau),({}_wY,\nu)$ and $({}_wZ,\omega)$ be m-topological spaces. If $f:({}_wX,\tau)\to({}_wY,\nu)$ and $g:({}_wY,\nu)\to({}_wZ,\omega)$ are m-continuous, then the mapping $g\circ f:({}_wX,\tau)\to({}_wZ,\omega)$ is m-continuous.

Proof: Let $P\in\omega$. Since g,f are m-continuous mappings, it follows that $g^{-1}(P)\in\nu$ and $f^{-1}[g^{-1}(P)]\in\tau$. Now, for each $x\in X$,

$$C_{(g\circ f)^{-1}(P)}(x)=C_P[gf](x)=C_P\big[g[f(x)]\big]=C_{g^{-1}(P)}[f(x)]=C_{f^{-1}[g^{-1}(P)]}(x)=C_{[f^{-1}\circ g^{-1}](P)}(x).$$

Thus, $(g\circ f)^{-1}(P)=f^{-1}\big[g^{-1}(P)\big]\in\tau$. Therefore, the mapping $g\circ f:({}_wX,\tau)\to({}_wZ,\omega)$ is m-continuous.

4. Conclusion and future work

In this paper, we have introduced a new approach to m-topological spaces and some of their important properties are studied. We have seen that in this m-topological spaces, the constant mapping is not m-continuous. But these properties are essential for developing algebraico topological structures. So, we have defined enriched m-topological spaces where constant mapping is m-continuous. The concepts of topological structures and their generalizations is one of the most powerful notions in branches of science and information systems. It is the generalized methods for measuring the similarity and dissimilarity between the objects in m-sets as universe. In this sense, and in developing the algebraico topological structures, this work has a great importance. There is an ample scope for further research to extend it in topological group theory which have many applications in abstract integration theory viz. Haar measure, Haar integral etc. and also in manifold theory through the development of Lie groups.

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