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Deteriorating Fuzzy Multi-Item Inventory Model with Shortages under Partially Backlogging for Power Demand Pattern

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Abstract

In this paper we have considered deteriorating fuzzy multi-item inventory model with shortages under partially backlogging. We consider, demand is power demand pattern and deterioration is proportional with time. Here the cost parameters are expressed as pentagonal fuzzy number. Next to solve multi-item inventory model by maxima or minima methods in second order partial derivatives of two variables and also using FNLP (Fuzzy Non-linear Programming) and FAGP (Fuzzy Additive Goal Programming) method. Finally, the model is illustrated by numerically example.

Keywords: *Inventory model, partially backlogging, Power demand pattern, Shortages, Deterioration, Pentagonal fuzzy number*

1. Introduction

In inventory model, deterioration and demand are important part and backlogging are complimentary phenomena. When the production is highly perishable, the reseller may need to backlog demand in order to certain costs due to deterioration which is defined as change, damage, decay, spoilage, obsolescence, evaporation, pilferage and loss of utility etc. The inventory is the physical stock of goods of any kind of item, which is kept by a businessman for smooth and efficient running of future aspects. The inventory controls play a role to take significant decisions that how much should be stocked and when should be ordered for stocks. The research work on inventory model has been tremendously expanded. In most of the inventory model, researchers assumed that the items could be stored indefinitely for future.

Bellman and Zadeh (1970) introduced decision-making in fuzzy environment. Dutta, Chakraborty & Roy (2007) gave the concept of an inventory model for single-period products with reordering opportunities under fuzzy demand. Das, Das & Mondal (2015) introduced an integrated production inventory model under interactive fuzzy credit period for deteriorating items with several markets. Halim, Giri and Chaudhuri (2009) gave the concept of fuzzy inventory models for an imperfect production system. Dutta & Kumar (2013) introduced fuzzy inventory model for deteriorating items with shortages under fully backlogged condition. Liu (1999) introduced Fuzzy criterion models for inventory systems with partial backorders. Mishra, Singh & Kumar (2013) gave the concept of an

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inventory model for deteriorating items with time-dependent demand and time-varying holding cost under partial backlogging.

In inventory model demand is power demand pattern and deterioration is proportional with time. Here, shortages are permitted and partial backlogging. The model is solved by maxima or minima methods in second order partial derivatives of two variables and also using FNLP (Fuzzy Non-linear Programming) and FAGP (Fuzzy Additive Goal Programming) method and finally the model is illustrated by numerical examples.

2. Notations and Assumptions

The following notations and assumptions are made in developing model:

Notations:

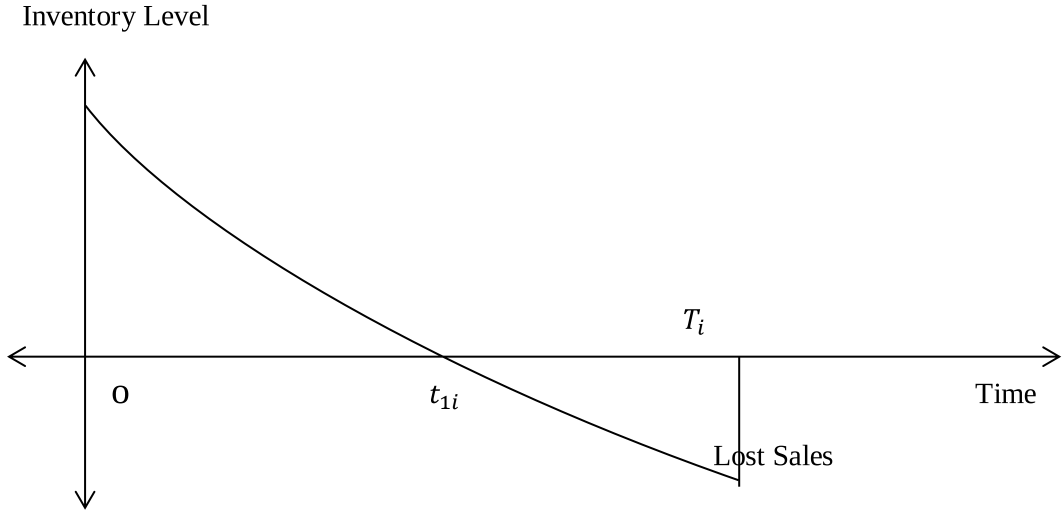
A_i : the set-up cost per unit time for i-th item.
h_i: the holding cost per unit time for i-th item.
c_{1i}: the shortages cost per unit time for i-th item.
c_{2i}: the lost sale cost per unit time for i-th item.
c_{3i}: the unit cost for i-th item.
α_i: the backlogging rate for i-th item, $0 \leq \alpha_i \leq 1$.
$I_{1i}(t_i)$: the level of positive inventory in the time interval $[0, t_{1i}]$ for i-th item.
$I_{2i}(t_i)$: the level of negative inventory in the time interval $[t_{1i}, T_i]$ for i-th item.

Assumption:

(i) The inventory deals multi-item.
(ii) Demand rate = $\frac{d_i t_i^{\frac{1-n}{n}}}{n T_i^{\frac{1}{n}}}$ which is power demand at time t_i, where d_i is constant.
(iii) Deterioration rate is proportional with time.
(iv) $\theta_i(t_i) = \theta_i t_i$, where θ_i is rate of deterioration, $0 < \theta_i < 1$.
(v) Replenishment is instantaneous, lead time is zero.

3. Mathematical Model

Figure 1. Graphical representation of inventory model



The inventory level decreases in $[0, t_{1i}]$ and becomes zero at t_{1i} and also the interval $[t_{1i}, T_i]$ is the shortages interval.

The differential equation during the period $[0, t_{1i}]$ and $[t_{1i}, T_i]$ are described as follows

$$\frac{dI_{1i}(t_i)}{dt_i} + \theta_i\, t_i\, I_{1i}(t_i) = -\frac{d_i t_i^{\frac{1-n}{n}}}{n T_i^{\frac{1}{n}}} \qquad \text{(for i = 1, 2,…, n)} \qquad \text{…… (1)}$$

and

$$\frac{dI_{2i}(t_i)}{dt_i} = -\,\alpha\, \frac{d_i t_i^{\frac{1-n}{n}}}{n T_i^{\frac{1}{n}}} \qquad \text{(for i = 1, 2,…, n)} \qquad \text{………… (2)}$$

Now using the boundary conditions, $I_{1i}(t_{1i}) = 0$ and $I_{2i}(t_{1i}) = 0$, then solving the differential equation (1) and (2) reduces to

$$I_{1i}(t_i) = \frac{d_i}{T_i^{\frac{1}{n}}}\left[\frac{n\theta_i}{(2n+1)}\left(t_i^{2+\frac{1}{n}} - t_{1i}^{2+\frac{1}{n}}\right) - \left(t_i^{\frac{1}{n}} - t_{1i}^{\frac{1}{n}}\right)\right] \qquad \text{………… (3)}$$

$$I_{2i}(t_i) = \frac{\alpha_i\, d_i}{T_i^{\frac{1}{n}}}\left(t_i^{\frac{1}{n}} - t_{1i}^{\frac{1}{n}}\right) \qquad \text{……………… (4)}$$

Set-up Cost: The set-up cost per unit time for i-th item is given by

$$SC_i = A_i \qquad \text{…………………. (5)}$$

Holding Cost: The inventory holding cost per unit time for i-th item is given by

$$HC_i = \int_0^{t_{1i}} h_i\, I_{1i}(t_i)\, dt_i$$

$$= h_i \frac{d_i}{T_i^{\frac{1}{n}}}\left[\frac{1}{(n+1)}\, t_{1i}^{\frac{1}{n}+1} - \frac{n\theta_i\,(2n+1)}{(2n+1)(3n+1)}\, t_{1i}^{\frac{1}{n}+3}\right] \qquad \text{………….. (6)}$$

Shortages Cost: The shortages cost per unit time for i-th item during the interval $[t_{1i}, T_i]$ given by

$$STC_i = c_{1i} \int_{t_{1i}}^{T_i} \{-I_{2i}(t_i)\}\, dt_i$$

$$= c_{1i} \frac{\alpha_i\, d_i}{T_i^{\frac{1}{n}}}\left[\frac{n}{(n+1)}\left(T_i^{\frac{1}{n}+1} - t_{1i}^{\frac{1}{n}+1}\right) - t_{1i}^{\frac{1}{n}}\,(T_i - t_{1i})\right] \qquad \text{………….. (7)}$$

Lost Sale Cost: The lost sale cost per unit time during the time interval $[t_{1i}, T_i]$ given by

$$LSC_i = c_{2i} \int_{t_{1i}}^{T_i} \{\, (1-\alpha_i)\, \frac{d_i t_i^{\frac{1-n}{n}}}{n T_i^{\frac{1}{n}}}\; dt_i$$

$$= c_{2i}\, \frac{(1-\alpha_i)\, d_i}{T_i^{\frac{1}{n}}}\left(T_i^{\frac{1}{n}} - t_{1i}^{\frac{1}{n}}\right) \qquad \text{……………………. (8)}$$

Purchase Cost: The purchase cost per unit time for i-th item is

$$PC_i = c_{3i}\left(I_{1i}(0) + \int_0^{t_{1i}} \alpha_i\, \frac{d_i t_i^{\frac{1-n}{n}}}{n T_i^{\frac{1}{n}}}\; dt_i\right)$$

$$= c_{3i} \, [\, (t_{1i}^{\frac{1}{n}} - \frac{n\theta_i}{(2n+1)} t_{1i}^{2+\frac{1}{n}}) + \frac{\alpha_i d_i}{T_i^{\frac{1}{n}}} (T_i^{\frac{1}{n}} - t_{1i}^{\frac{1}{n}})\,] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Therefore, the total average cost per unit time for i-th item is given by

$$\text{TA}C_i\,(t_{1i},T_i) = \frac{1}{T_i}\,[\text{S}C_i + \text{H}C_i + \text{ST}C_i + \text{LS}C_i + \text{P}C_i\,]$$

$$= \frac{1}{T_i}\,[A_i + h_i \frac{d_i}{T_i^{\frac{1}{n}}} \{\frac{1}{(n+1)} t_{1i}^{\frac{1}{n}+1} - \frac{n\theta_i\,(2n+1)}{(2n+1)(3n+1)} t_{1i}^{\frac{1}{n}+3}\} + c_{1i} \frac{\alpha_i d_i}{T_i^{\frac{1}{n}}} \{\frac{n}{(n+1)} (T_i^{\frac{1}{n}+1} - t_{1i}^{\frac{1}{n}+1}) - t_{1i}^{\frac{1}{n}} (T_i - t_{1i})\} + c_{2i} \frac{(1-\alpha_i)\, d_i}{T_i^{\frac{1}{n}}} (T_i^{\frac{1}{n}} - t_{1i}^{\frac{1}{n}}) + c_{3i}\, \{\, (t_{1i}^{\frac{1}{n}} - \frac{n\theta_i}{(2n+1)} t_{1i}^{2+\frac{1}{n}}) + \frac{\alpha_i d_i}{T_i^{\frac{1}{n}}} (T_i^{\frac{1}{n}} - t_{1i}^{\frac{1}{n}})\,\}]$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Now using Khun-Tuckar condition, differentiating partially with respect to t_{1i},T_i in equation (10), then its follows that

$$\frac{\partial}{\partial t_{1i}} \text{TA}C_i\,(t_{1i},T_i) \quad \text{and} \quad \frac{\partial}{\partial T_i} \text{TA}C_i\,(t_{1i},T_i)$$

For maximization or minimization,

$$\frac{\partial}{\partial t_{1i}} \text{TA}C_i\,(t_{1i},T_i) = 0 \quad \text{and} \quad \frac{\partial}{\partial T_i} \text{TA}C_i\,(t_{1i},T_i) = 0. \quad \ldots\ldots\ldots (11)$$

From the above equation we have the optimal value of t_{1i},T_i and the total cost is minimum providing the equation satisfied the following conditions

$$\left(\frac{\partial^2}{\partial t_{1i}^2} (\text{TA}C_i\,(t_{1i},T_i))\right) \left(\frac{\partial^2}{\partial T_i^2} (\text{TA}C_i\,(t_{1i},T_i))\right) - \frac{\partial^2}{\partial t_{1i}\partial T_i} (\text{TA}C_i\,(t_{1i},T_i)) > 0.$$

and

$$\left(\frac{\partial^2}{\partial t_{1i}^2} (\text{TA}C_i\,(t_{1i},T_i))\right) > 0.$$

Therefore, we will obtain total average minimum cost for i-th item when we put the optimal value of t_{1i},T_i in equation (10).

4. Mathematical Analysis

Pentagonal Fuzzy Number (PFN):

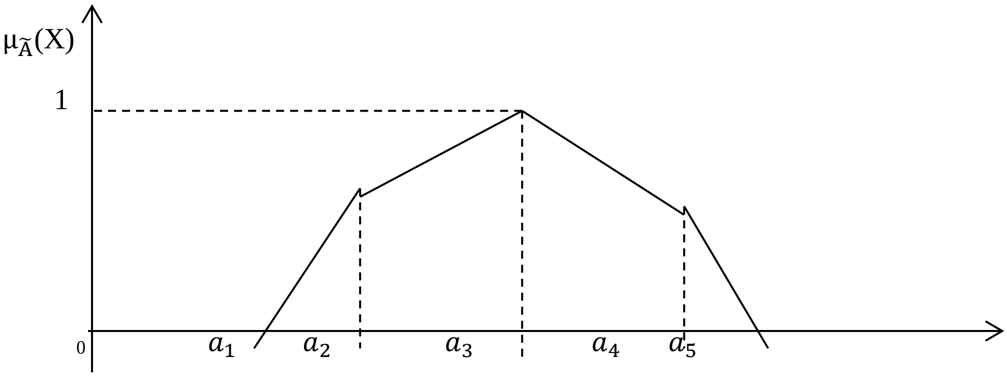
A pentagonal fuzzy number A is denoted as $\widetilde{A} = (a_1, a_2, a_3, a_4, a_5)$, where a_3 is the middle point and (a_1, a_2) and (a_4, a_5) are the left and right side points of a_3, respectively. Now, we construct the mathematical definition of a pentagonal fuzzy number.

Definition: A fuzzy number $\widetilde{A} = (a_1, a_2, a_3, a_4, a_5)$ is called a pentagonal fuzzy number when the membership function has the form

$$\mu_{\tilde{A}}(X) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ \frac{x-a_2}{a_3-a_2}, & a_2 \le x \le a_3 \\ 1 & x = a_3 \\ \frac{a_4-x}{a_4-a_3}, & a_3 \le x \le a_4 \\ \frac{a_5-x}{a_5-a_4}, & a_4 \le x \le a_5 \end{cases}$$

and geometrical figure of pentagonal fuzzy number

Figure 2. Pentagonal fuzzy number



Some properties of pentagonal fuzzy number (PFN):

Addition: Let $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5)$ be two PFNs; then,

$\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5)$.

Subtraction: Let $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5)$ be two PFNs; then,

$\tilde{A} - \tilde{B} = (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4, a_5 - b_5)$.

Multiplication: Let $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5)$ be two PFNs; then,

$\tilde{A}\ \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4, a_5 b_5)$.

Scalar Multiplication: Let $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$ be PFN and $k \in R$ be any scalar. If $k \geq 0$,

k$\tilde{A} = (ka_1, ka_2, ka_3, ka_4, ka_5)$.

Division: Let $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5)$ be two PFNs; then,

$\frac{\tilde{A}}{\tilde{B}} = (\frac{a_1}{b_5}, \frac{a_2}{b_4}, \frac{a_3}{b_3}, \frac{a_4}{b_2}, \frac{a_5}{b_1})$.

Graded mean integration method (Definition):

The graded mean h-level value of generalized fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, W_A)$ is given by $\frac{h(L^{-1}(h)+R^{-1}(h))}{2}$, where L^{-1} and R^{-1} are the inverse function of L and R respectively. Then graded mean integration of pentagonal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$ defined as follows

P $(\tilde{A}) = \int_0^{W_A)} \frac{h(L^{-1}(h)+R^{-1}(h))}{2}$ dh $/\int_0^{W_A} h\, dh$, where $0 < h \le W_A$ and $0 < W_A \le 1$.

5. Fuzzy Model

Due to uncertainty cost let us assume that

$\tilde{A}_i = (A_{11}, A_{12}, A_{13}, A_{14}, A_{15})$, $\tilde{h}_i = (h_{11}, h_{12}, h_{13}, h_{14}, h_{15})$, $\tilde{c}_{1i} = (c_{11}, c_{12}, c_{13}, c_{14}, c_{15})$, $\tilde{c}_{2i} = (c_{21}, c_{22}, c_{23}, c_{24}, c_{25})$, $\tilde{c}_{3i} = (c_{31}, c_{32}, c_{33}, c_{34}, c_{35})$, be the pentagonal fuzzy number.

The total average cost is given by

$$\widetilde{TAC_i}\,(t_{1i},T_i) = \frac{1}{T_i}\,[\mathrm{S}C_i + \mathrm{H}C_i + \mathrm{ST}C_i + \mathrm{LS}C_i + \mathrm{P}C_i\,]$$

$$= \frac{1}{T_i}\,[\tilde{A}_i + \tilde{h}_i\,\frac{d_i}{T_i^{\frac{1}{n}}}\,\{\frac{1}{(n+1)}\,t_{1i}^{\frac{1}{n}+1} - \frac{n\theta_i\,(2n+1)}{(2n+1)(3n+1)}\,t_{1i}^{\frac{1}{n}+3}\,\} + \tilde{c}_{1i}\,\frac{\alpha_i\,d_i}{T_i^{\frac{1}{n}}}\,\{\frac{n}{(n+1)}\,(T_i^{\frac{1}{n}+1} - t_{1i}^{\frac{1}{n}+1}) - t_{1i}^{\frac{1}{n}}\,(T_i - t_{1i})\,\} + \tilde{c}_{2i}\,\frac{(1-\alpha_i)\,d_i}{T_i^{\frac{1}{n}}}\,(T_i^{\frac{1}{n}} - t_{1i}^{\frac{1}{n}}) + \tilde{c}_{3i}\,\{\,(t_{1i}^{\frac{1}{n}} - \frac{n\theta_i}{(2n+1)}\,t_{1i}^{2+\frac{1}{n}}) + \frac{\alpha_i\,d_i}{T_i^{\frac{1}{n}}}\,(T_i^{\frac{1}{n}} - t_{1i}^{\frac{1}{n}})\,\}]$$

The total average cost $\widetilde{TAC_i}\,(t_{1i},T_i)$ by graded mean representation method as follows

$$\widetilde{TAC_i}\,(t_{1i},T_i) = \frac{1}{12}\,[\widetilde{TAC_i^1}\,(t_{1i},T_i), \widetilde{TAC_i^2}\,(t_{1i},T_i), \widetilde{TAC_i^3}\,(t_{1i},T_i), \widetilde{TAC_i^4}\,(t_{1i},T_i), \widetilde{TAC_i^5}\,(t_{1i},T_i)]$$

Where

$$\widetilde{TAC_i^k}\,(t_{1i},T_i) = \frac{1}{T_i}\,[\mathrm{S}C_i + \mathrm{H}C_i + \mathrm{ST}C_i + \mathrm{LS}C_i + \mathrm{P}C_i\,]$$

$$= \frac{1}{T_i}\,[\tilde{A}_i^k + \tilde{h}_i^k\,\frac{d_i}{T_i^{\frac{1}{n}}}\,\{\frac{1}{(n+1)}\,t_{1i}^{\frac{1}{n}+1} - \frac{n\theta_i\,(2n+1)}{(2n+1)(3n+1)}\,t_{1i}^{\frac{1}{n}+3}\,\} + \tilde{c}_{1i}^k\,\frac{\alpha_i\,d_i}{T_i^{\frac{1}{n}}}\,\{\frac{n}{(n+1)}\,(T_i^{\frac{1}{n}+1} - t_{1i}^{\frac{1}{n}+1}) - t_{1i}^{\frac{1}{n}}\,(T_i - t_{1i})\,\} + \tilde{c}_{2i}^k\,\frac{(1-\alpha_i)\,d_i}{T_i^{\frac{1}{n}}}\,(T_i^{\frac{1}{n}} - t_{1i}^{\frac{1}{n}}) + \tilde{c}_{3i}^k\,\{\,(t_{1i}^{\frac{1}{n}} - \frac{n\theta_i}{(2n+1)}\,t_{1i}^{2+\frac{1}{n}}) + \frac{\alpha_i\,d_i}{T_i^{\frac{1}{n}}}\,(T_i^{\frac{1}{n}} - t_{1i}^{\frac{1}{n}})\,\}].$$

k = 1, 2, 3, 4, 5

For minimum cost it should be

$$\frac{\partial}{\partial t_{1i}}\widetilde{TAC_i}\,(t_{1i},T_i) = 0 \quad \text{and} \quad \frac{\partial}{\partial T_i}\widetilde{TAC_i}\,(t_{1i},T_i) = 0.$$

and

$$(\frac{\partial^2}{\partial t_{1i}^2}\,(\widetilde{TAC_i}\,(t_{1i},T_i)\,)\,)\,(\frac{\partial^2}{\partial T_i^2}\,(\widetilde{TAC_i}\,(t_{1i},T_i)\,)\,) - \frac{\partial^2}{\partial t_{1i}\partial T_i}\,(\widetilde{TAC_i}\,(t_{1i},T_i)) > 0.$$

and

$$(\frac{\partial^2}{\partial t_{1i}^2}\,(\widetilde{TAC_i}\,(t_{1i},T_i)\,)\,) > 0.$$

6. Numerical Example

For crisp model:

Table 1. Input Data

Items	A_i	h_i	d_i	n	θ_i	α_i	c_{1i}	c_{2i}	c_{3i}
1	100	0.9	25	2	0.4	0.3	10	7	20
2	90	0.8	20	3	0.5	0.2	15	6	18

The results is obtained from table-2

Table-2

Items	t_{1i}	T_i	TAC_i
1	1.215	1.782	2247.82
2	1.314	1.825	2425.21

From equation (10) and using FNLP and FAGP methods we have in table-3

Table-3

Method	Item	t_{1i}	T_i	TAC_i
FNLP	1	1.201	1.702	2194.85
	2	1.248	1.756	2328.42
FAGP	1	1.287	1.727	2127.24
	2	1.304	1.784	2298.02

For change of deterioration, we obtain optimal values and total minimum cost

Table-4

θ_i	Items	t_{1i}	T_i	TAC_i
0.01	1	1.028	1.341	1902
	2	1.037	1.372	1987
0.02	1	1.038	1.355	1950
	2	1.057	1.387	2009
0.03	1	1.049	1.368	1991
	2	1.065	1.392	2047
0.04	1	1.061	1.381	2015
	2	1.074	1.393	2077
0.05	1	1.070	1.392	2045
	2	1.085	1.402	2098

For fuzzy model:

Table 5. input fuzzy number

Items	$\tilde{A}_i$	$\tilde{h}_i$	$\tilde{c}_{1i}$	$\tilde{c}_{2i}$	$\tilde{c}_{3i}$
1	(80,90,100,110,120)	(0.7,0.8,0.9,1,1.1)	(6,8,10,12,14)	(5,6,7,8,9)	(16,18,20,22,24)
2	(80,85,90,95,100)	(0.6,0.7,0.8,0.9,1)	(11,13,15,17,19)	(4,5,6,7,8)	(14,16,18,20,22)

Now, using graded mean representation, the solution of fuzzy model is

(i) When $\tilde{A}_i$, $\tilde{h}_i$, $\tilde{c}_{1i}$, $\tilde{c}_{2i}$, $\tilde{c}_{3i}$ are pentagonal fuzzy number then the solution of fuzzy model

Table-6

Items	t_{1i}	T_i	TAC_i
1	1.217	1.725	2245.21
2	1.316	1.822	2423.83

(ii) When $\tilde{A}_i$, $\tilde{h}_i$, $\tilde{c}_{1i}$, $\tilde{c}_{2i}$ are pentagonal fuzzy number then the solution of fuzzy model

Table-7

Items	t_{1i}	T_i	TAC_i
1	1.216	1.725	2244.52
2	1.315	1.882	2423.81

(iii) When $\tilde{A}_i$, $\tilde{h}_i$, $\tilde{c}_{1i}$ are pentagonal fuzzy number then the solution of fuzzy model

Table-8

Items	t_{1i}	T_i	TAC_i
1	1.217	1.725	2245.23
2	1.315	1.823	2423.51

(iv) When $\tilde{A}_i$, $\tilde{h}_i$ are pentagonal fuzzy number then the solution of fuzzy model

Table-9

Items	t_{1i}	T_i	TAC_i
1	1.215	1.724	2246.21
2	1.316	1.824	2424.87

(v) When $\tilde{A}_i$ is pentagonal fuzzy number then the solution of fuzzy model

Table-10

Items	t_{1i}	T_i	TAC_i
1	1.217	1.727	2246.31
2	1.315	1.823	2425.28

The observation from table-4, for change of deterioration (θ_i) we obtain optimal values of t_{1i}, T_i and minimum total cost. Now for crisp fuzzy model and fuzzy model from table 2, 5, 6, 7, 8, 9 and 10, we obtain optimal values of t_{1i}, T_i (for two items) and with the help of total inventory cost function of the model providing total inventory cost per unit time for the inventory system is minimum.

7. Conclusion

In this paper, we have proposed a real life inventory problem in fuzzy inventory model which is developed by multi-item inventory model with shortages under partial backlogging for power demand pattern. The model is solved numerically by maxima or minima methods in second order partial derivatives of two variables and also using FNLP (Fuzzy Non-linear Programming) and FAGP (Fuzzy Additive Goal Programming) method. Finally, the model has been verified by numerically example and the result indicates validity and stability of the model.

8. Acknowledgements

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References

Mishra, V. K., Singh, L.S. & Kumar, R. (2013). An inventory model for deteriorating items with time-dependent demand and time-varying holding cost under partial backlogging, *Journal of Industrial Engineering International*, 9:4.

Mandal, W. A. & Islam, S. (2015). Fuzzy inventory model for power demand pattern with inlation, shortages under partial backlogging, *International Journal of Science and Research*, 4(7).

Zimmermann, H. J. (1978). Fuzzy programming and linear programming with several objective functions, *Fuzzy set and Systems*, l(1), pp. 45-55.

Huang, Y. F. & Chung, K. J. (2013). Optimal replenishment and payment policies in EOQ model under cash discount and trade credit, *Asia-Pacific Journal of Operational Research*, 20:1,77-90.

Liang, Y. & Zhou, F. (2011). A two warehouse inventory model for deteriorating items under conditionally permissible delay in payment, *Applied Mathematical Model*, 35, 2221-2231.

Zadeh, L. A. (1965). Fuzzy sets, *Information and Control*, 8, 338-353.

Halim, K. A., Giri, B. C. & Chaudhuri, K. S. (2009). Fuzzy inventory models for an imperfect production system, *International Journal of System Science*, 40:1,45-52.

Liu, B. (1999). Fuzzy criterion models for inventory systems with partial backorders, *Annals of Operations Research*, 87, 117-126.

Ukil, S. I. & Uddin, Md. S. (2016). A production inventory model of constant production rate and demand of level dependent linear trend, *American Journal of Operations Research*, 6, 61-70.

Kumar, S. & Rajput, U. S. (2015). Fuzzy inventory model for deteriorating items with time dependent demand and partial backlogging, *Applied Mathematics*, 6, 496-509.

Dutta, D. & Kumar, P. (2013). Fuzzy inventory model for deteriorating items with shortages under fully backlogged condition, *International Journal of Soft Computing and Engineering*, 3(2).

Sharmila, D. & Kumar, R. U. (2015). Inventory model for deteriorating items involving fuzzy with shortages and exponential demand, *International Journal of Supply and Operations Management*, 2(3), pp 888-904.

Das, B. C., Das, B. & Mondal, S. K. (2015). An integrated production inventory model under interactive fuzzy credit period for deteriorating items with several markets, *Applied Soft Computing* 28, 453-465.

Dutta, P., Chakraborty, D. & Roy, A. R. (2007). An inventory model for single-period products with reordering opportunities under fuzzy demand, *Computers and Mathematics with Applications* 53,1502-1517.

Pal, S., Mahapatra, G. S. & Samanta, G. P. (2015). A production inventory model for deteriorating item with ramp type demand allowing inflation and shortages under fuzziness, *Economic Modelling*, 46, 334-345.

Bellman, R. E. & Zadeh, L. A. (1970). Decision-making in Fuzzy Environment, *Management Science*, 17(4) B 141- B 164.

Das, S & Islam, S. (2014). Multi-Objective inventory model for deteriorating items with shortages, A fuzzy programming approach, *Mathematical Manuscripts*, 7 (1), 15-24.

Tripathy, C.K. & Pradhan, L.M. (2010). An EOQ model for Weibull deteriorating items with power demand & partial backlogging, *International Journal of Contemporary Mathematical Sciences*, 5(38), 1895-1904.

Kumar, S. & Rajput, U.S. (2015). An inventory model for perishable items with time varying stock dependent demand and trade credit under inflation, *American Journal of Operations Research*, 5, 435-449.

Shah, N. H. & Shukla, K.T. (2009). Deteriorating inventory model for waiting time partial backlogging, *Applied Mathematical Sciences*, 3(9), 421-428.