Journal Homepage: www.katwacollegejournal.com

Two warehouse inventory model for deteriorating items and stock dependent demand under conditionally permissible delay in payment

Satya Kumar Das[1], Govt. General Degree College, Gopiballavpur-II, India
Sahidul Islam, University of Kalyani, Kalyani, India

Article Record: Received Jan. 12 2018, Revised paper received May 18 2018, Final Acceptance June 6 2018
Available Online June 7 2018

Abstract

In this paper, the own warehouse has limited capacity W and rented warehouse (RW) has unlimited capacity. After storing the W unit in OW, excess unit are storing in RW. The RW is assume to offer better preserving facilities than the OW resulting in a lower rate of deterioration and is assume to charge higher holding cost than the OW. In this model, two warehouses are considered for deteriorating items with stock dependent demand under conditionally permissible delay in payment. Numerical examples are given to demonstrate the theoretical results. Sensitivity analysis of the optimal solution with respect to major parameters of the system has been carried out the implication are discussed in detail. In the sensitivity analysis, suggestions are given to minimize the total cost of the inventory system.

Keywords: Inventory model, Two warehouses, Deterioration, Stock dependent demand, Permissible delay in payment

1. Introduction

Inventory management is one of the most challenging issues in the organization. In 1915, the first inventory model was developed by F.Harris (1915). In the business world, deterioration is an important key factor. In general, deterioration is defined as the damage, spoilage, dryness, vaporization etc. that results in decrease of usefulness of the original one. Inventory problems for deteriorating items have been widely studied Sarma(1987), Yang (2006), Rong (2008), Hsieh, Dye and Ouyang(2008), Huang(2003), Goyal&Giri(2001), Zhou & Yang(2005), Yang &chang(2013), Tayal, Singh & Sharma(2015). Goyal and Giri (2001) formulated a model on inventory of deteriorating inventory and Lee formulated a model on two warehouse inventory model with deteriorating under FIFO dispatching policy. Zhou(2003) Developed a multi warehouse inventory model for items with time varying demand and shortages. When suppliers provide price discount for bulk purchases or the products are seasonal, the retailers may purchase more goods that can be stored in own warehouse (OW). Therefore a rented warehouse (RW) is used to store the excess units over the fixed capacity W of the own warehouse. Generally, the rented warehouse is to charge higher unit holding cost than the own warehouse, but to offer a better preserving facility resulting in a lower rate of deterioration for the goods than the own warehouse. To reduce the inventory cost, it will be economical to consume the goods of RW at the earliest. Consequently, the firm

[1] Corresponding author

stores goods in OW before RW, but clears the stocks in RW before OW. Recently, the literature Zhou & Yang (2005) proposed a two warehouse inventory model for items with stock level dependent demand rate and Rong et al. Mahapatra &Maity(2008) developed an optimization inventory policy for a deteriorating item with imprecise lead time, partially/fully backlogged shortages and price dependent demand under two warehouse system. The most related paper is Liang and Zhou (2011).
Besides, the assumption of constant demand is not always applicable to real situations. For instant, it is usually observed in the supermarket that display of the consumer goods in large quantities attracts more customers and generates higher demand. In the last several years, many researchers have given considerable attention to the situation where the demand rate is dependent on the level of the on hand inventory. Gupta and Vart (1986) were first to develop models for stock dependent consumption rate. The related papers are also Bhunia &Maity (1998), Singh & Sharma (2015), Yang &Chang(2013).
In the traditional economic order quantity (EOQ) model, it often assumed that the retailer must pay off as soon as the items are received. In fact, the supplier offers the retailer a delay period, known as trade credit period, in the paying for purchasing cost, which is very common business practice. During the trade credit period the retailer can accumulate revenues by selling items and by earning interests. In the research field Goyal(1985) was the first to establish an EOQ model with a constant demand rate under the condition of permissible delay in payments.

In this study a two warehouse inventory model for deteriorating items is developed in which demand is stock dependent and delay in payment is permitted. In that model assume that the RW has higher unit holding cost than the OW. The purpose of this study is to make the model more relevant and applicable in the practice. The proposed mathematical model converges to prerivous models such as in Goyal (1985), Aggarwal and Jaggi(1995) and Yanlai Liang, Fangming Zhou (2011) when consider the special case.

The rest of the paper is organized as follows: In Section 1, the introduction and in Section 2, the notations and section 3, the assumptions which are used throughout this article, are descried. In section 4, the formulation of the model to minimize the total annual inventory cost is established. Section 5 illustrates the numerical examples. Sensitivity analyses of the optimal solution with respect to major parameters are carried out in section 6. Finally, conclusions are draw and the future researches are pointed out in section 7.

Mathematical model:

2. Notations

$D(t)$: Demand rate at time t.

$I(t)$: Inventory level at time t

A: Replenishment cost (Ordering cost) for replacement the items.

p: Selling price/unit.

c : The purchasing cost/unit.

h_0: The holding cost/unit time in own house.

h_r: The holding cost/unit time in rented warehouse.

α: Deterioration in own house.

β : Deterioration in rented warehouse.

M : Retailer trade credit period offered by the supplier.

I_e: Interest earn by the retailer (dollar/year).

I_p: Interest payable to the retailer (dollar/year).

W : Stock capacity Inventoryin own house.

t_w: Time length in which the inventory level reduced to W.

T: Length of the order cycle.
$I_r(t)$: Inventory level at time t in rented warehouse and obviously $0 \leq t \leq t_w$
$I_0(t)$: Inventory level at time t in own house and obviously $0 \leq t \leq T$
$I_R(t)$: Total numbers of sells inventory item at time t where $t \leq t_w$
$I_{O,R}(t)$: Total numbers of sells inventory item attime t where$t_w < t \leq T$
TC : The average relevant inventory cost per unit time of the inventory systems.

3. Assumptions

1) Replenishment rate is infinite and lead time is zero.
2) Shortages are not allowed.
3) The inventory system considers only one item.
4) The stoke dependent demand rate $D(t) = a + bI(t)$, where a and b are positive constants and$I(t)$is the inventory level at time t.
5) The own house (OW) has limited capacity of W units rent warehouse (RW) has unlimited capacity. For economic the items of RW are consumed first and next the items of OW.
6) $p > c$
7) $h_0 < h_r$
8) The rent warehouse offers better facility, so $\alpha > \beta$
9)$h_r - h_0 > c(\alpha - \beta)$
10) We assume that the maximum deteriorating quantity for items in own house, αW is less than the demand rate D(t), that is $\alpha W < D(t)$.
11) The retailer can accumulate revenue and earn interest after his/her customer pays for the amount of purchasing cost to the retailer until the end of the trade credit period offered by the supplier.

4. Formulation of the model

At time t, the inventory level in RW and OW is satisfying the following differential equation :

$$\frac{dI_r(t)}{dt} = -D(t) - \beta I_r(t), \qquad 0 \leq t \leq t_w.$$

$$= -a - bI_r(t) - bI_0(t) - \beta I_r(t) \quad (1)$$

With the boundary condition

$$I_r(t_w) = 0 \qquad (2)$$

And $\frac{dI_0(t)}{dt} = -\alpha I_0(t), \qquad 0 \leq t \leq t_w.$ (3)

With the boundary condition $I_0(0) = W$ (4)

While during the interval $[t_w, T]$, the inventory level in OW, $I_0(t)$, is governed by the following differential equation.

$$\frac{dI_0(t)}{dt} = -D(t) - \alpha I_0(t)$$

$$= -a - bI_0(t) - \alpha I_0(t) \text{ when } t_w \leq t \leq T \qquad (5)$$

With the boundary condition $I_0(T) = 0$ (6)

From (3) and (4) we have

$$I_0(t) = We^{-\alpha t}, \text{ when } 0 \leq t \leq t_w \qquad (7)$$

From (5) and (6) we have

$$I_0(t) = \frac{a}{b+\alpha}\left[e^{(b+\alpha)(T-t)} - 1\right] \text{ where } t_w \leq t \leq T \qquad (8)$$

From (1) and (2) we have

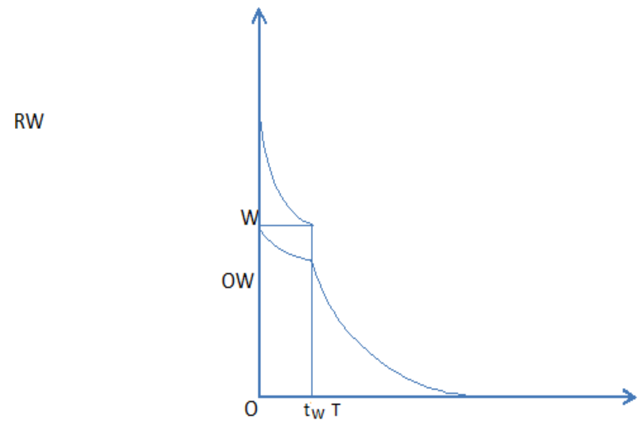
$I_r(t) = -\left[\frac{a}{b+\beta} + \frac{bW}{b+\beta-\alpha}e^{-\alpha t}\right] + e^{(b+\beta)(t_w-t)}\left[\frac{a}{b+\beta} + \frac{bW}{b+\beta-\alpha}e^{-\alpha t_w}\right]$ where $0 \le t \le t_w$ (9)

Consider the continuity of $I_0(t)$ at $t = t_w$ i.e

$$\frac{a}{b+\alpha}\left[e^{(b+\alpha)(T-t_w)} - 1\right] = We^{-\alpha t_w}$$

$$T = t_w + \frac{1}{b+\alpha} ln\left[1 + \frac{W(b+\alpha)}{a}e^{-\alpha t_w}\right] \quad (10)$$

Figure 1. Graphical representation of two-warehouse inventory system



Based on the assumptions and notations we obtain the following annual relevant cost as follows:

(i) The annual ordering cost $= \frac{A}{T}$

(ii) Annual stock holding cost:

Holding cost in rented warehouse is $= \frac{h_r}{T}\int_0^{t_w} I_r(t)dt$

$$= \frac{h_r}{T}\int_0^{t_w}\left[-\left[\frac{a}{b+\beta} + \frac{bW}{b+\beta-\alpha}e^{-\alpha t}\right] + e^{(b+\beta)(t_w-t)}\left[\frac{a}{b+\beta} + \frac{bW}{b+\beta-\alpha}e^{-\alpha t_w}\right]\right]dt$$

$$= \frac{h_r}{T}\left[\frac{-a}{(b+\beta)}t_w + \frac{bW}{\alpha(b+\beta-\alpha)}(e^{-\alpha t_w} - 1) - \left\{\frac{a}{(b+\beta)^2} + \frac{bW}{(b+\beta-\alpha)(b+\beta)}e^{-\alpha t_w}\right\}\left(1 - e^{(b+\beta)t_w}\right)\right] \quad (11)$$

Holding cost in own house is $= \frac{h_0}{T}\left[\int_0^{t_w} I_0(t)dt + \int_{t_w}^{T} I_0(t)dt\right]$

$$= \frac{h_0}{T}\left[\int_0^{t_w} We^{-\alpha t}dt + \int_{t_w}^{T}\frac{a}{b+\alpha}\left[e^{(b+\alpha)(T-t)} - 1\right]dt\right]$$

$$= \frac{h_0}{T}\left[\frac{W}{\alpha}(1 - e^{-\alpha t_w}) - \frac{a}{(b+\alpha)^2}\left(1 - e^{(b+\alpha)(T-t_w)}\right) - \frac{a}{b+\alpha}(T - t_w)\right] \quad (12)$$

Therefore the total annual stock holding cost is

$$= \frac{h_r}{T}\left[\frac{-a}{(b+\beta)}t_w + \frac{bW}{\alpha(b+\beta-\alpha)}(e^{-\alpha t_w} - 1) - \left\{\frac{a}{(b+\beta)^2} + \frac{bW}{(b+\beta-\alpha)(b+\beta)}e^{-\alpha t_w}\right\}\left(1 - e^{(b+\beta)}\right)\right] + \frac{h_0}{T}\left[\frac{W}{\alpha}(1 - e^{-\alpha t_w}) - \frac{a}{(b+\alpha)^2}\left(1 - e^{(b+\alpha)(T-t_w)}\right) - \frac{a}{b+\alpha}(T - t_w)\right] \quad (13)$$

(iii) Annual deteriorating cost :

Number of deteriorating items in rented warehouse is

$$= \int_0^{t_w} \beta I_r(t)dt$$

$= \beta \left[\frac{-a}{(b+\beta)} t_w + \frac{bW}{\alpha(b+\beta-\alpha)} (e^{-\alpha t_w} - 1) - \left\{\frac{a}{(b+\beta)^2} + \frac{bW}{(b+\beta-\alpha)(b+\beta)} e^{-\alpha t_w}\right\} \left(1 - e^{(b+\beta)t_w}\right)\right]$ (14)

Number of deteriorating items in own warehouse is

$$= \alpha \left[\int_0^{t_w} I_0(t)dt + \int_{t_w}^{T} I_0(t)dt\right]$$

$= \alpha \left[\frac{W}{\alpha}(1 - e^{-\alpha t_w}) - \frac{a}{(b+\alpha)^2}\left(1 - e^{(b+\alpha)(T-t_w)}\right) - \frac{a}{b+\alpha}(T - t_w)\right]$ (15)

Annual total deteriorating cost is

$= \frac{c}{T}\left[\alpha \left\{\frac{W}{\alpha}(1 - e^{-\alpha t_w}) - \frac{a}{(b+\alpha)^2}\left(1 - e^{(b+\alpha)(T-t_w)}\right) - \frac{a}{b+\alpha}(T - t_w)\right\} + \beta \left\{\frac{-a}{(b+\beta)} t_w + \frac{bW}{\alpha(b+\beta-\alpha)} (e^{-\alpha t_w} - 1) - \left\{\frac{a}{(b+\beta)^2} + \frac{bW}{(b+\beta-\alpha)(b+\beta)} e^{-\alpha t_w}\right\} \left(1 - e^{(b+\beta)t_w}\right)\right\}\right]$ (16)

Figure 2. Trade credit period for case 1 i.e $M \leq t_w < T$

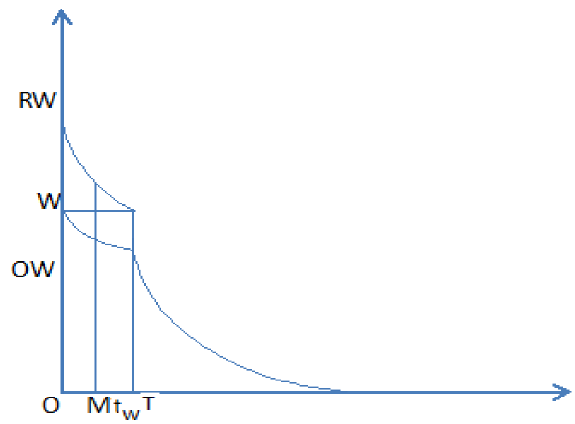


Figure 3. Trade credit period for case 2 i.e$t_w < M \leq T$

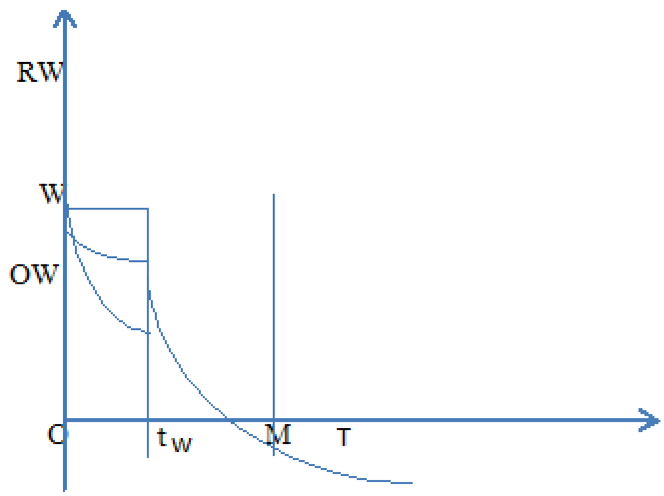
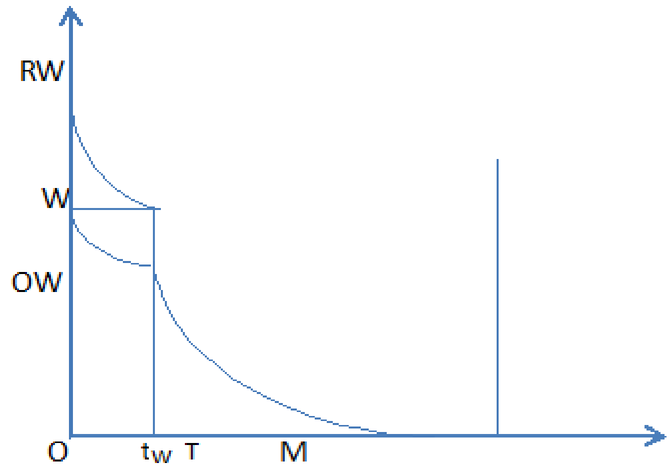


Figure 4. Trade credit period for case 3 i.e $t_w < T < M$



(iv)The interest payable opportunity cost :

There are three cases may arises as following

case 1: $M \leq t_w < T$

In this case, the annual interest payable is

$$= \frac{cI_p}{T}\left[\int_M^{t_w} I_r(t)dt + \int_M^{T} I_0(t)dt\right]$$

$$= \frac{cI_p}{T}\left[\frac{a}{(b+\beta)}(M-t_w) + \frac{bW}{\alpha(b+\beta-\alpha)}(e^{-\alpha t_w} - e^{-\alpha M}) + \left\{\frac{a}{(b+\beta)^2} + \frac{bW}{(b+\beta-\alpha)(b+\beta)}e^{-\alpha t_w}\right\}\left(e^{(b+\beta)(t_w-M)} - 1\right) - \frac{W}{\alpha}(e^{-\alpha t_w} - e^{-\alpha M}) - \frac{a}{(b+\alpha)^2}\left(1 - e^{(b+\alpha)(T-t_w)}\right) - \frac{a}{b+\alpha}(T-t_w)\right]$$

(17)

case 2:$t_w < M \leq T$

In this case, the annual interest payable is

$$= \frac{cI_p}{T}\int_M^T I_0(t)dt$$

$$= \frac{acI_p}{T(b+\alpha)}\left[\frac{1}{b+\alpha}\left(e^{(b+\alpha)(T-M)} - 1\right) - (T-M)\right] \quad (18)$$

Case 3: $t_w < T < M$

In this case no interest charges are paid.

(v)Different cases of opportunity interest earned:

The total sells items at time t $(t \leq t_w)$is

$$I_R(t) = \int_0^t D(t)dt$$

$$= \int_0^t \{a + bI_0(t) + bI_r(t)\}dt$$

$$= at + \frac{bW}{\alpha}(1 - e^{-\alpha t}) - b\left[\left\{\frac{at}{b+\beta} - \frac{bW}{\alpha(b+\beta-\alpha)}(e^{-\alpha t} - 1)\right\} - e^{(b+\beta)t_w}\left\{\frac{a}{(b+\beta)^2} + \frac{bW}{(b+\beta-\alpha)(b+\beta)}e^{-\alpha t_w}\right\}\left(1 - e^{-(b+\beta)t}\right)\right] \quad (19)$$

The total sells items at time t $(t_w < t)$are

$$I_{O,R}(t) = \int_0^t D(t)dt$$

$$= \int_0^{t_w} D(t)dt + \int_{t_w}^t D(t)dt$$

$$= at + \frac{bW}{\alpha}(1 - e^{-\alpha t_w}) - b\left[\left\{\frac{a}{b+\beta}t_w - \frac{bW}{\alpha(b+\beta-\alpha)}(e^{-\alpha t_w} - 1)\right\} - e^{(b+\beta)t_w}\left\{\frac{a}{(b+\beta)^2} + \frac{bW}{(b+\beta-\alpha)(b+\beta)}e^{-\alpha t_w}\right\}\left(1 - e^{-(b+\beta)t_w}\right)\right] - \frac{ab}{(b+\alpha)}\left[\frac{1}{b+\alpha}\left(e^{(b+\alpha)(T-t)} - e^{(b+\alpha)(T-t_w)}\right) - (t - t_w)\right] \quad (20)$$

when$t_w \le t < T$

There two cases may arise as following :

case 1:$M \le t_w < T$

In this case the annual interest earn is

$$= \frac{pI_e}{T}\int_0^M I_R(t)dt$$

$$= \frac{pI_e}{T}\int_0^M \left\{at + \frac{bW}{\alpha}(1 - e^{-\alpha t}) - b\left[\left\{\frac{at}{b+\beta} - \frac{bW}{\alpha(b+\beta-\alpha)}(e^{-\alpha t} - 1)\right\} - e^{(b+\beta)t_w}\left\{\frac{a}{(b+\beta)^2} + \frac{bW}{(b+\beta-\alpha)(b+\beta)}e^{-\alpha t_w}\right\}\left(1 - e^{-(b+\beta)t}\right)\right]\right\}dt$$

$$= \left[\frac{aM^2}{2} + \frac{bW}{\alpha}\left(M + \frac{e^{-\alpha M}}{\alpha} - \frac{1}{\alpha}\right) - b\left\{\left(\frac{aM^2}{2(b+\beta)} + \frac{bW}{\alpha(b+\beta-\alpha)}\left(M + \frac{e^{-\alpha M}}{\alpha} - \frac{1}{\alpha}\right)\right) - e^{(b+\beta)t_w}\left\{\frac{a}{(b+\beta)^2} + \frac{bW}{(b+\beta-\alpha)(b+\beta)}e^{-\alpha t_w}\right\}\left\{M + \frac{1}{b+\beta}\left(e^{-(b+\beta)M} - 1\right)\right\}\right\}\right]\frac{pI_e}{T} \quad (21)$$

case 2 :$t_w < M \le T$

In this case the annual interest earn is

$$= \frac{pI_e}{T}\int_0^M I_{O,R}(t)dt$$

$$= \frac{pI_e}{T}\int_0^M \left[at + \frac{bW}{\alpha}(1 - e^{-\alpha t_w}) - b\left[\left\{\frac{a}{b+\beta}t_w - \frac{bW}{\alpha(b+\beta-\alpha)}(e^{-\alpha t_w} - 1)\right\} - e^{(b+\beta)t_w}\left\{\frac{a}{(b+\beta)^2} + \frac{bW}{(b+\beta-\alpha)(b+\beta)}e^{-\alpha t_w}\right\}\left(1 - e^{-(b+\beta)t_w}\right)\right] - \frac{ab}{(b+\alpha)}\left[\frac{1}{b+\alpha}\left(e^{(b+\alpha)(T-t)} - e^{(b+\alpha)(T-t_w)}\right) - (t - t_w)\right]\right]dt$$

$$= \frac{pI_e}{T}\left[\frac{aM^2}{2} + \frac{bWM}{\alpha}(1 - e^{-\alpha t_w}) - bM\left[\left\{\frac{a}{b+\beta}t_w - \frac{bW}{\alpha(b+\beta-\alpha)}(e^{-\alpha t_w} - 1)\right\} - e^{(b+\beta)t_w}\left\{\frac{a}{(b+\beta)^2} + \frac{bW}{(b+\beta-\alpha)(b+\beta)}e^{-\alpha t_w}\right\}\left(1 - e^{-(b+\beta)t_w}\right)\right] + \frac{ab}{(b+\alpha)^3}\left\{e^{(b+\alpha)(T-M)} - e^{(b+\alpha)T}\right\} + \frac{abM}{(b+\alpha)^2}e^{(b+\alpha)(T-t_w)} + \frac{ab}{(b+\alpha)}\left(\frac{M^2}{2} - Mt_w\right)\right] \quad (22)$$

case 3 : $t_w < T < M$

In this case the annual interest earn is

$$= \frac{pI_e}{T}\left[\int_0^T I_{O,R}(t)dt + (\mathrm{M}-\mathrm{T})\, I_{O,R}(T)\right]$$

$$= \frac{pI_e}{T}\Big[aT\left(M-\frac{T}{2}\right)+\frac{bWM}{\alpha}(1-e^{-\alpha t_w})-bM\Big[\Big\{\frac{a}{b+\beta}t_w-\frac{bW}{\alpha(b+\beta-\alpha)}(e^{-\alpha t_w}-1)\Big\}-e^{(b+\beta)t_w}\Big\{\frac{a}{(b+\beta)^2}+\frac{bW}{(b+\beta-\alpha)(b+\beta)}e^{-\alpha t_w}\Big\}\left(1-e^{-(b+\beta)t_w}\right)\Big]-\frac{ab(M-T)}{(b+\alpha)^2}+\frac{abM}{(b+\alpha)^2}e^{(b+\alpha)(T-t_w)}+\frac{ab}{(b+\alpha)^3}\left(1-e^{(b+\alpha)T}\right)+\frac{ab}{(b+\alpha)}\left(MT-Mt_w-\frac{1}{2}T^2\right)\Big] \qquad (23)$$

Therefore the total relevant cost for the retailer can be expressed as

$TC(t_w,T)$ =< Ordering cost > + < Stock holding cost in RW> + < Stock holding cost in OW > + < Inventory payable opportunity cost > + < Deterioration cost> - <Opportunity interest earn >.

$$TC(t_w,T) = TC_1\ , \quad if \quad M \le t_w < T$$
$$= TC_2\ , \quad if \quad t_w < M \le T$$
$$= TC_2\ , \quad if \quad t_w < T < M$$

Where $TC_1 = \frac{A}{T}+\frac{h_r}{T}\Big[\frac{-a}{(b+\beta)}t_w+\frac{bW}{\alpha(b+\beta-\alpha)}(e^{-\alpha t_w}-1)-\Big\{\frac{a}{(b+\beta)^2}+\frac{bW}{(b+\beta-\alpha)(b+\beta)}e^{-\alpha t_w}\Big\}\left(1-e^{(b+\beta)}\right)\Big]+\frac{h_0}{T}\Big[\frac{W}{\alpha}(1-e^{-\alpha t_w})-\frac{a}{(b+\alpha)^2}\left(1-e^{(b+\alpha)(T-t_w)}\right)-\frac{a}{b+\alpha}(T-t_w)\Big]+\frac{c}{T}\Big[\alpha\Big\{\frac{W}{\alpha}(1-e^{-\alpha t_w})-\frac{a}{(b+\alpha)^2}\left(1-e^{(b+\alpha)(T-t_w)}\right)-\frac{a}{b+\alpha}(T-t_w)\Big\}+\beta\Big\{\frac{-a}{(b+\beta)}t_w+\frac{bW}{\alpha(b+\beta-\alpha)}(e^{-\alpha t_w}-1)-\Big\{\frac{a}{(b+\beta)^2}+\frac{bW}{(b+\beta-\alpha)(b+\beta)}e^{-\alpha t_w}\Big\}\left(1-e^{(b+\beta)t_w}\right)\Big\}\Big]+\frac{cI_p}{T}\Big[\frac{a}{(b+\beta)}(M-t_w)+\frac{bW}{\alpha(b+\beta-\alpha)}(e^{-\alpha t_w}-e^{-\alpha M})+\Big\{\frac{a}{(b+\beta)^2}+\frac{bW}{(b+\beta-\alpha)(b+\beta)}e^{-\alpha t_w}\Big\}\left(e^{(b+\beta)(t_w-M)}-1\right)-\frac{W}{\alpha}(e^{-\alpha t_w}-e^{-\alpha M})-\frac{a}{(b+\alpha)^2}\left(1-e^{(b+\alpha)(T-t_w)}\right)-\frac{a}{b+\alpha}(T-t_w)\Big]-\frac{pI_e}{T}\Bigg[\frac{aM^2}{2}+\frac{bW}{\alpha}\left(M+\frac{e^{-\alpha M}}{\alpha}-\frac{1}{\alpha}\right)-b\Bigg\{\left(\frac{aM^2}{2(b+\beta)}+\frac{bW}{\alpha(b+\beta-\alpha)}\left(M+\frac{e^{-\alpha M}}{\alpha}-\frac{1}{\alpha}\right)\right)-e^{(b+\beta)t_w}\Big\{\frac{a}{(b+\beta)^2}+\frac{bW}{(b+\beta-\alpha)(b+\beta)}e^{-\alpha t_w}\Big\}\Big\{M+\frac{1}{b+\beta}\left(e^{-(b+\beta)M}-1\right)\Big\}\Bigg\}\Bigg]$ for $M \le t_w < T$.

$TC_2 = \frac{A}{T}+\frac{h_r}{T}\Big[\frac{-a}{(b+\beta)}t_w+\frac{bW}{\alpha(b+\beta-\alpha)}(e^{-\alpha t_w}-1)-\Big\{\frac{a}{(b+\beta)^2}+\frac{bW}{(b+\beta-\alpha)(b+\beta)}e^{-\alpha t_w}\Big\}\left(1-e^{(b+\beta)}\right)\Big]+\frac{h_0}{T}\Big[\frac{W}{\alpha}(1-e^{-\alpha t_w})-\frac{a}{(b+\alpha)^2}\left(1-e^{(b+\alpha)(T-t_w)}\right)-\frac{a}{b+\alpha}(T-t_w)\Big]+\frac{c}{T}\Big[\alpha\Big\{\frac{W}{\alpha}(1-e^{-\alpha t_w})-\frac{a}{(b+\alpha)^2}\left(1-e^{(b+\alpha)(T-t_w)}\right)-\frac{a}{b+\alpha}(T-t_w)\Big\}+\beta\Big\{\frac{-a}{(b+\beta)}t_w+\frac{bW}{\alpha(b+\beta-\alpha)}(e^{-\alpha t_w}-1)-\Big\{\frac{a}{(b+\beta)^2}+\frac{bW}{(b+\beta-\alpha)(b+\beta)}e^{-\alpha t_w}\Big\}\left(1-e^{(b+\beta)t_w}\right)\Big\}\Big]+\frac{\alpha cI_p}{T(b+\alpha)}\Big[\frac{1}{b+\alpha}\left(e^{(b+\alpha)(T-M)}-1\right)-(T-M)\Big]-\frac{pI_e}{T}\Big[\frac{aM^2}{2}+\frac{bWM}{\alpha}(1-e^{-\alpha t_w})-bM\Big[\Big\{\frac{a}{b+\beta}t_w-\frac{bW}{\alpha(b+\beta-\alpha)}(e^{-\alpha t_w}-1)\Big\}-e^{(b+\beta)t_w}\Big\{\frac{a}{(b+\beta)^2}+\frac{bW}{(b+\beta-\alpha)(b+\beta)}e^{-\alpha t_w}\Big\}\left(1-e^{-(b+\beta)t_w}\right)\Big]+\frac{ab}{(b+\alpha)^3}\Big\{e^{(b+\alpha)(T-M)}-e^{(b+\alpha)T}\Big\}+\frac{abM}{(b+\alpha)^2}e^{(b+\alpha)(T-t_w)}+\frac{ab}{(b+\alpha)}\left(\frac{M^2}{2}-Mt_w\right)\Big]$ for $t_w < M \le T$. $TC_3 = \frac{A}{T}+\frac{h_r}{T}\Big[\frac{-a}{(b+\beta)}t_w+\frac{bW}{\alpha(b+\beta-\alpha)}(e^{-\alpha t_w}-1)-\Big\{\frac{a}{(b+\beta)^2}+\frac{bW}{(b+\beta-\alpha)(b+\beta)}e^{-\alpha t_w}\Big\}\left(1-e^{(b+\beta)}\right)\Big]+\frac{h_0}{T}\Big[\frac{W}{\alpha}(1-e^{-\alpha t_w})-\frac{a}{(b+\alpha)^2}\left(1-e^{(b+\alpha)(T-t_w)}\right)-\frac{a}{b+\alpha}(T-t_w)\Big]+\frac{c}{T}\Big[\alpha\Big\{\frac{W}{\alpha}(1-e^{-\alpha t_w})-\frac{a}{(b+\alpha)^2}\left(1-e^{(b+\alpha)(T-t_w)}\right)-\frac{a}{b+\alpha}(T-t_w)\Big\}+\beta\Big\{\frac{-a}{(b+\beta)}t_w+$

$\frac{bW}{\alpha(b+\beta-\alpha)}(e^{-\alpha t_w}-1)-\left\{\frac{a}{(b+\beta)^2}+\frac{bW}{(b+\beta-\alpha)(b+\beta)}e^{-\alpha t_w}\right\}\left(1-e^{(b+\beta)t_w}\right)\Big\}\Big]-\frac{pI_e}{T}\left[aT\left(M-\frac{T}{2}\right)+\right.$
$\frac{bWM}{\alpha}(1-e^{-\alpha t_w})-bM\left[\left\{\frac{a}{b+\beta}t_w-\frac{bW}{\alpha(b+\beta-\alpha)}(e^{-\alpha t_w}-1)\right\}-e^{(b+\beta)t_w}\left\{\frac{a}{(b+\beta)^2}+\right.\right.$
$\left.\frac{bW}{(b+\beta-\alpha)(b+\beta)}e^{-\alpha t_w}\right\}\left(1-e^{-(b+\beta)t_w}\right)\Big]-\frac{ab(M-T)}{(b+\alpha)^2}+\frac{abM}{(b+\alpha)^2}e^{(b+\alpha)(T-t_w)}+\frac{ab}{(b+\alpha)^3}\left(1-e^{(b+\alpha)T}\right)+$
$\left.\frac{ab}{(b+\alpha)}\left(MT-Mt_w-\frac{1}{2}T^2\right)\right]$

for $t_w < T < M$.

When, $b=0$, TC is the same as Yanlani Liang and Fangming Zhou's [19] model and also when $b=0, t_w=0, \alpha=\beta=0$ and $W\to\infty$ then TC is the same as Goyal's [20] model. In addition, when $h_r=h_0$ and $W\to\infty$, TC is the same as Aggarwal and Jaggi's [21] model.

5. Numerical Example

To illustrate the model a numerical example has been solved.

Example 1 : Consider an inventory system with the following parameters

A=2000$/order, c=20$/unit,p=35$/unit, I_p=0.15$/year,I_e==0.12$/year, M=0.33 year, W=100units, a=100 units, b=0.25, α=0.1, β=0.06, h_r=3 $/unit/year, h_0=1 $/unit/year.

According to the above parameters the optimal solution is t_w^* =0.9079 year, T^* =2.8004 years ,TC^*=1313.06$. It is clear that the inventory in rented warehouse vanish at 0.9079 year and then after 1.8925 years both rented warehouse and own warehouse are empty. Minimum cost in this system is 1313.06$.

Example 2 : Same data of example 1 put in case 2 i.e TC_2 then we get optimal solution t_w^*=0.2803 year , T^*=2.1567 years , TC^*=1574.76$.

Example 3 : Same data of example 1 put in case 3 i.e TC_3 then we get infeasible solution t_w^*=0.0000 year , T^*=0.3300 year , TC^*=5939.79$.

Example 4 : Consider another inventory system with the following parameters

A=500$/order, c=25$/unit,p=35$/unit, I_p=0.15$/year,I_e=0.12$/year, M=0.25 year, W=200 units, a=100 units, b=0.25, α=0.1, β=0.06, h_r=3 $/unit/year, h_0=2 $/unit/year.

According to the above parameters the optimal solution is t_w^* =0.2500 year, T^* =1.2359 years, TC^*=862.48$. It is clear that the inventory in rented warehouse vanishes at 0.2500 year andthen after 1.2359 years both rented warehouse and own warehouse are empty. Minimum cost inthis system is 862.48$.

Example 5: Same data of example 4 put in case 2i.eTC_2 then we get optimal solution
t_w^*= 0.0000 year,T^* =0.9551 year, TC^*=841.34$.

Example 6 : Same data of example 4 put in case 3 i.eTC_3 then we get infeasible solution
t_w^*=0.0000 year, T^* =0.2500 year, TC^*=1921.04$.

Above examples suggest that case one is the most suitable for the minimum inventory cost in business.All the above examples have been solved by LINGO13 and the optimal solutions are displayedin table-I.

TABLE I: Optimal solutions of different examples.

Example	t_w^*(years)	T^*(years)	TC^* ($)	Remarks

1	0.9079	2.8004	1313.06	-
2	0.2803	2.1567	1574.76	-
3	0.0000	0.3300	5939.79	Infeasible
4	0.2500	1.2359	862.48	-
5	0.0000	0.9551	841.34	-
6	0.0000	0.2500	1921.04	Infeasible

6. Sensitivity Analysis

TABLE II: Effect of changes in the system parameters of the model

W(units)	a	A($/order)	b	t_w^*	T^*	TC^*
100	100	2000	0.25	0.9079	2.8004	1312.06
			0.30	0.9056	2.7264	1353.81
			0.35	0.8971	2.6566	1387.87
		2500	0.25	1.0646	3.1205	1481.87
			0.30	1.0547	3.0302	1527.44
			0.35	1.0394	2.9402	1566.27
		3000	0.25	1.2008	3.3979	1635.24
			0.30	1.1841	3.2929	1685.53
			0.35	1.1627	3.1958	1729.03
200	200	2000	0.25	0.3883	2.3929	1427.84
			0.30	0.4148	2.3453	1475.93
			0.35	0.4301	2.2956	1515.44
		2500	0.25	0.5723	2.7574	1621.87
			0.30	0.5865	2.6862	1674.54
			0.35	0.5914	2.6168	1718.85
		3000	0.25	0.7274	3.3606	1793.60
			0.30	0.7314	2.9728	1851.17
			0.35	0.7276	2.8869	1900.46
300	300	2000	0.25	0.3300	2.3746	1469.85
			0.30	0.3300	2.2993	1520.28
			0.35	0.3300	2.2332	1561.91
		2500	0.25	0.3300	2.5603	1672.41
			0.30	0.3300	2.4748	1729.66
			0.35	0.3300	2.3997	1777.74
		3000	0.25	0.3300	2.7240	1861.60
			0.30	0.3300	2.6290	1925.53
			0.35	0.3300	2.5455	1979.89

Above table described the following:

(a) When the ordering cost A is increased, both the optimal cycle time T^* and relevant total costs TC^* will increase simultaneously. This implies that the retailer may order more quantity toreduce the average total relevant costs.

(b) When retailer's warehouse capacity W is increasing the optimal replenishment cycle time T^* will decrease but the relevant total costs TC^*will be increased. This implies that the retailer canorder quantity less frequent to reduce costs when retailer own largest storage space.

(c) When the demand is increasing, the optimal cycle time T^* is decreasing but relevant total costs $TC^*$are increasing. This implies that the retailer may order less quantity to take the benefitsof the trade credit more frequently.

7. Conclusions

In this article, a two warehouse inventory model has been proposed for deteriorating items with stock dependent demand under permissible delay in payment. Shortages are not permitted in this system. Numerical examples are illustrated this inventory system. Here also assume that the rented warehouse charges higher unit holding cost than the own warehouse. Our aim was to find the optimal replenishment policies for minimizing the total relevant inventory costs. The proposed model can be used in inventory control of certain non-instantaneous deteriorating items such that food items, electronic components, fashionable commodities and others. Furthermore, sensitivity analysis was carried out with respect to the key parameters and useful managerial insights were obtained.

In the future study, it is hoped to further incorporate the proposed model into more realistic assumption, such as probabilistic demand, introduce shortages, generalize the model under two-level credit period strategy.

Acknowledgements

The authors are thankful to University of Kalyani for providing financial assistance through DST-PURSE Programme. The authors are grateful to the reviewers for their comments and suggestions.

References

Aggarwal, S.P. & C.K.Jaggi, (1995). Ordering policies of deteriorating items under permissible delay in payments, *Journal of the Operational Research Society*, 46, 658-662

Bunia, A.K. & Maiti, M. (1998). Deterministic inventory model for deteriorating items with finite rate of replenishment dependent on inventory level, *Computers and Operations Research*, 25, 997-1006

Goyal, S.K. & Giri, B.C. (2001). Recent trends in modeling of deteriorating inventory, *European Journal of Operational Research*, 134, 1-16

Goyal, S.K. (1985). Economic order quantity under conditions of permissible delay in payments, *Journal of the Operational Research Society*, 36, 35-38

Gupta,R. & Vrat , P. (1986). Inventory model with multi-items under constraint systems for stock dependent consumption rate, *Operations Research* 24, 41-42

Harris, F. (1915). Operation and cost, A.W Shaw Co. Chicago

Hsieh, T.P., Dye, C.Y. & Ouyang, L.Y. (2008). Determining optimal lot size for a two warehouse system with deterioration and shortages using net present value, *European Journal of Operational Research*, 191, 182-192

Huang, Y.F. (2003). An inventory model under two levels of trade credit andlimited shortage space derived without derivatives, *Applied mathematical modeling*, 27, 983-996

Liang, Y. & Zhou, F. (2011). A two-warehouse inventory model for deteriorating items under conditionally permissible delay in payment, *Applied Mathematical modeling* 35, 2221-2231

Ouyang, L.Y., Wu, K.S. & Yang, C.Te. (2006). A study on an inventory model for non-instantaneous deterioration items with permissible delay in payments, *Computers and Industrial Engineering*, 51, 637-651

Rong, M., Mahapatra, N.K. & Maiti, M. (2008). A two warehouse inventory model for deteriorating item with partially/fully backlogged shortage and fuzzy lead time, *European Journal of Operational Research*, 189, 59-75

Salameh, M.K., Abboud, N.E., Kassar, A.N.El- & Ghattas, R.E. (2003). Continuous review inventory model with delay in payments, *International Journal of Production Economics*, 85, 91-95

Sarma, K.V.S (1987). A deterministic order level inventory model for deteriorating items with two storage facilities, *European Journal of Operational Research*, 29, 70-73

Tayal S, Singh S R, Sharma S (2015) An inventory model for deteriorating items with seasonal products and an option of an alternative market. Uncertain Supply Chain Management 3: 69-86.

Yang HL, Chang CT (2013) A two-warehouse partial backlogging inventory model for deteriorating items with permissible delay in payment under inflation, *Applied Mathematical Modelling*, 37:2717–2726.

Yang, H. L. (2006). Two-warehouse partial backlogging inventory model for deteriorating items under inflation, *International Journal of Production Economics*, 103, 362-370

Zhou, Y.W. & Yang, S.L. (2005). A two warehouse inventory model for items with stock level dependent demand rate, *International Journal of Production Economics*, 95, 215-228

Zhou, Y.W. (2003). A multi warehouse inventory model for items with time varying demand and shortages, *Computers and Operations Research*, 30, 2115-2131